Compensation of self-focusing distortions in quasiresonant amplification of a light pulse

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A theoretical analysis is made of the compensation of a positive nonlinear refractive index of insulators containing impurities by saturation of the refractive index in the anomalous dispersion region. The exact solutions are obtained for self-similar amplification of a laser pulse under conditions of homogeneous broadening of a resonant transition. It is shown that profiling of the time envelope of a steady-state bleaching wave of a saturable absorber together with detuning of the pulse frequency upward from resonance makes it possible to increase the threshold of small-scale self-focusing.

INTRODUCTION

Self-focusing distortions of the wave pattern are among the main reasons for the increase in the divergence of laser radiation.[1] The physical cause of such distortions is the dependence of the refractive index on the radiation intensity. Two mechanisms of this optical nonlinearity can be identified in the case of solid-state amplifiers: nonresonant, when the nonlinear component of the refractive index is proportional to the intensity,[2] and resonant, which appears in the case of a small frequency detuning from the center of the gain profile.[3] It is important to note that in the latter case the sign of the nonlinearity is governed by the sign of the detuning $\Delta\omega$ of the radiation frequency from a resonance and, depending on this sign, the nonlinearity can be focusing or defocusing (Fig. 1). As pointed out in Refs. 4 and 5, it follows that a suitable selection of the magnitude and sign of the detuning make it possible to compensate linearity of the refractive index. However, a quantitative analysis given in Refs. 4 and 5 was limited to the case of a constant population inversion, which is generally not true of laser amplifying media; moreover, no analysis was made of the stability in the presence of perturbations of the transverse spatial structure. The case of weak saturation was considered in Ref. 6, where an analysis was made of the conditions for the compensation of a thermal self-focusing lens in a ruby amplifier due to a resonant refractive index and it was assumed that the population inversion decreases proportionally to the transmitted pulse energy $\int_{-\infty}^{t} I(z,\theta)\,d\theta$. We shall obtain exact solutions describing mutual compensation of self-focusing distortions without assuming that the saturation of the active medium is weak. We shall show that the spatial stability of the resultant solutions is governed by the kinetics of the amplification process.

1. MUTUAL COMPENSATION CONDITIONS

Propagation of a light pulse of duration $T_1 \gg \tau_p \gg T_2$ in a solid-state laser amplifier characterized by a homogeneous broadening of the spectral line is described by the following equation[7] if the linear losses are ignored (and the amplifier length is less than a limiting value):

$$\frac{\partial \mathscr{E}}{\partial z} + \frac{1}{v}\frac{\partial \mathscr{E}}{\partial t} + \frac{i\Delta_\perp \mathscr{E}}{2k} = \frac{\sigma N(z, t=0)}{2}\mathscr{E}(1 + i\Delta\omega T_2)\exp\left[-2\sigma\int_{-\infty}^{t} I(z,\theta)\,d\theta\right] + ikn_2 I(z,t)\mathscr{E}. \quad (1)$$

Here, $\mathscr{E}$ is the amplitude of the electric field; v is the group velocity; I is the intensity; $N(z, 0)$ is the distribution of the population inversion $N(z, t)$ at the initial moment; T_2^{-1} is the half-width of the spectral line; T_1 is the longitudinal relaxation time; k is the wave number; $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the transverse Laplacian; $n_2 I$ is the nonresonant nonlinear refractive index due to the electron or electron–nuclear nonlinearity of the insulator base of the active medium.[1] We shall analyze the solutions of Eq. (1) for small values of the detuning $\Delta\omega T_2 \ll 1$, when $\sigma = \sigma_0/[1 + (\Delta\omega T_2)^2]$ is close to its value σ_0 at the line center.

We shall seek the solution in the form of a plane wave ($\Delta_\perp \mathscr{E} = 0$). We shall first consider the case of complete compensation of the resonant refractive index, i.e., we shall assume that the imaginary terms on the right-hand side of Eq. (1) balance each other out:

$$\tfrac{1}{2}\,\sigma N(z,0)\,\Delta\omega T_2 \exp\left(-2\sigma\int_{-\infty}^{t} I(z,\theta)\,d\theta\right) = kn_2 I(z,t). \quad (2)$$

The solution of Eq. (2) is the time dependence of the intensity:

$$I(z,t) = a(z)/2\sigma(a(z)t+1); \quad a(z) = 2\sigma^2 N(z,0)\Delta\omega T_2/n_2 k. \quad (3)$$

It is clear from this solution that the time profile of a pulse should be matched to the spatial profile of the population inversion $N(z, 0)$. The latter is found from Eq. (1) naturally on condition that $I(z, t)$ is described by Eq. (3). Since in this case (and also because $\Delta_\perp \mathscr{E} = 0$) there is no phase modulation, we can find $N(z, t = 0)$ using the Frantz–Nodvik equation[8]

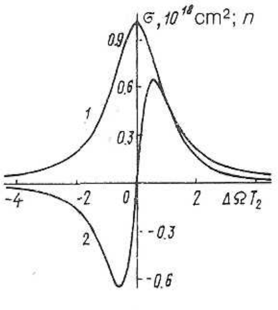

FIG. 1. Amplification cross section σ (1) and resonant refractive index n (2) near the center of a homogeneously broadened spectral line.

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$$\frac{\partial I}{\partial z}+\frac{1}{v}\frac{\partial I}{\partial t}=\sigma N(z,0)I\exp\left(-2\sigma\int_{-\infty}^{t}I(z,\theta)d\theta\right), \quad (4)$$

the solution of which is well known[7]:

$$I(z,t)=\frac{I(0,t)}{1+\exp\left(-2\sigma\int_0^{t-z/v}I(z,\theta)d\theta\right)\left(\exp\left(-\sigma\int_0^z N(\xi,0)d\xi\right)-1\right)}. \quad (5)$$

It contains the initial inversion profile as an unknown function which now has to be selected so that the condition (3) is satisfied. This selection is possible at any moment in time provided $\tau_p \gg nl/c$ (l is the amplifier length and n is the linear refractive index) and it gives the distribution of the population inversion at the initial moment:

$$N(z,0)=N(0,0)/[1-\sigma N(0,0)z]. \quad (6)$$

Knowing the time dependence of the intensity given by Eq. (3), we can readily find the inversion at an arbitrary moment t:

$$N(z,t)=N(0,0)/[a(0)t+1-\sigma N(0,0)z]. \quad (7)$$

Therefore, the mutual compensation of the nonresonant and resonant components of the refractive index can be ensured by creating at the initial moment a population inversion distribution obeying Eq. (6) and by injecting into an amplifier (at $z=0$) a pulse of the type described by Eq. (3). A special feature of these initial conditions is an increase in $N(z,t)$ with z and the increase in the inversion N to infinity in a finite distance $z_\infty=[\sigma N(0,0)]^{-1}$. This circumstance limits the amplifier length by the condition $l<z_\infty$.

The physical meaning of this behavior of the population inversion is that an increase in $N(z,t)$ increases the resonant refractive index [$\sigma N(z,t)\Delta\omega T_2$ in Eq. (1)], which compensates the increase in the nonresonant refractive index [$n_2 I(z,t)$] due to an increase in the intensity in the course of propagation of a pulse along the amplifier. The shape of a pulse given by Eq. (3) is characterized by a fall of the intensity with time (Fig. 2). This is due to the saturation effect, which is responsible for the decrease of the resonant refractive index with time in each section z in accordance with $\exp(-2\sigma\int_{-\infty}^{t}I(z,\theta)d\theta)$.

We shall now consider compensation of just the variable part of the resonant refractive index which appears as a result of saturation of the laser transition. It is this part that creates a nontrivial phase modulation that alters the transverse structure of a pulse.[9]

We shall now rewrite Eq. (1) in the form

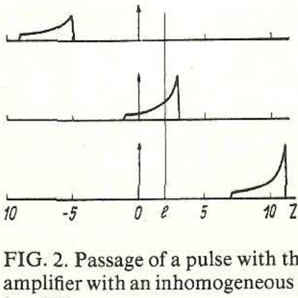

FIG. 2. Passage of a pulse with the profile described by Eq. (3) across an amplifier with an inhomogeneous distribution of the inversion obeying the law (7).

$$\frac{\partial \mathscr{E}}{\partial z}+\frac{1}{v}\frac{\partial \mathscr{E}}{\partial t}+\frac{i\Delta_\perp \mathscr{E}}{2k}=\sigma N(z,0)\mathscr{E}\exp\left[-2\sigma\int_{-\infty}^{t}I(z,\theta)d\theta\right]$$
$$-i\sigma N(z,0)\Delta\omega T_2\mathscr{E}\left\{1-\exp\left[-2\sigma\int_{-\infty}^{t}I(z,\theta)d\theta\right]\right\}$$
$$+i\sigma N(z,0)\Delta\omega T_2\mathscr{E}+in_2 I(z,t)k\mathscr{E}, \quad (8)$$

where the second term on the right-side of Eq. (8) is the variable part of the resonant refractive index, whereas the third term is the constant part (in the solutions this part gives rise to an intensity-independent phase factor which does not affect the transverse structure). The compensation equation is now

$$\sigma N(z,0)\Delta\omega T_2\left(1-\exp\left[-2\sigma\int_{-\infty}^{t}I(z,\theta)d\theta\right]\right)=kn_2 I(z,t), \quad (9)$$

the solution of which together with Eq. (4) gives

$$I(z,t)=a/2\sigma(1+\exp[-(\sigma N_0 z+at)]). \quad (10)$$

In contrast to Eq. (3), this solution is valid for an arbitrary relationship between τ_p and nl/c. A more general form of the solution is obtained from Eq. (10) by replacing the self-similar variable $\sigma N_0 z+at$ with $\sigma N_0 z+at-az/v$.

The difference between the solution (10) and the solutions (3) and (7) is as follows: firstly, the population inversion should be distributed uniformly along the amplifier [$N/(z,0)=N_0$]. Secondly, the solution (10) describes a fairly long "prepulse" (strictly speaking, this prepulse begins at $-\infty$), which is a mathematical consequence of the absence of nonzero solutions in Eq. (9) subject to the initial condition $I(t_H>-\infty)=0$ and can be understood quite readily from the physical point of view on the basis of the following considerations: since during the first moments after the arrival of a pulse in the amplifying medium ($t\gtrsim-\infty$) the quantity $\int_{-\infty}^{t}I(z,\theta)d\theta$ is practically zero, it follows that the variable part of the resonant refractive index cannot compensate the nonresonant refractive index if the latter is not sufficiently small.

An interesting feature of this case is the constancy of the pulse profile (i.e., its self-similarity) when the maximum intensity $I(z,+\infty)=a/2\sigma$ is constant for any value of z, in spite of the increase in the energy of a pulse during amplification. This is a consequence of the displacement of the intensity drop (inversion "burnup" front, see Fig. 3) along the prepulse and is illustrated clearly by an increase in the area between $t=-\infty$ and $t=0$. This behavior of the envelope is due to the familiar superradiant propagation effect.[7] We re-

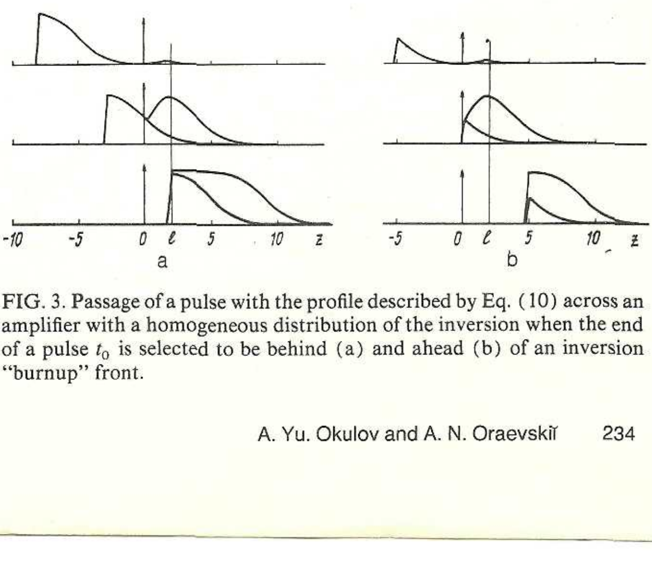

FIG. 3. Passage of a pulse with the profile described by Eq. (10) across an amplifier with a homogeneous distribution of the inversion when the end of a pulse t_0 is selected to be behind (a) and ahead (b) of an inversion "burnup" front.

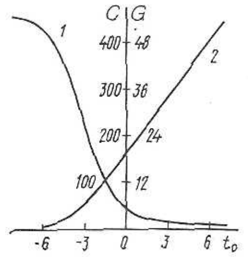

FIG. 4. Dependences of G (1) and of C (2) on the position of the moment of termination of the entry pulse t_0 relative to the midpoint of the inversion "burnup" front ($t = 0$).

call that in this case the amplifier length is less than its limiting value and a steady-state asymmetric pulse, generated because of the balance of nonlinear amplification and linear losses,[7] cannot form in the available time.

The gain is defined as the ratio of the energies in the amplified and freely traveling pulses (Fig. 3) lying between t_p and t_0:

$$G = \ln\left[\frac{\exp(\sigma N_0 l + at_0) + 1}{\exp(\sigma N_0 l + at_p) + 1}\right] \Big/ \ln\left[\frac{\exp(at_0) + 1}{\exp(at_p) + 1}\right].$$

A comparison of Figs. 3a and 3b shows that G increases as the final moment t_0 is shifted toward negative values (Fig. 3b), which is due to amplification of an increasing part of the pulse under weak saturation conditions (Fig. 4a). The contrast

$$C = \ln[\exp(\sigma N_0 l + at_0) + 1]/\ln[\exp(\sigma N_0 l + at_p) + 1] - 1,$$

defined (as usual) as the ratio of the energy in a pulse (between t_p and t_0) to the energy in the prepulse (up to t_p), falls on reduction in t_0 for a fixed value of t_p (Fig. 4b). Therefore, the selection of t_0 defines the gain G and the contrast C.

We shall obtain quantitative estimates for near-optimal dimensionless parameters of the problem $\sigma N_0 l = 4$, $at_{fr} = 2$, $st_{fr}/n > l$, where t_{fr} is the duration of the "burnup" front to the level 0.9. In the case of a YAG:Nd^{3+} crystal (with homogeneous broadening of the spectral line), we have $\sigma = (2\text{–}8)\times 10^{-19}$ cm^2 and $n = 1.5$, so that for $l = 4$ cm we obtain $N_0 = (5\text{–}1.2)\times 10^{18}$ cm^{-3}. If $t_{fr} = 10^{-9}$ s and $n_2 = 5\times 10^{-16}$ cm^2/W, we should have $\Delta\omega T_2 = 0.03\text{–}0.075$, so that when the line width is $T_2^{-1} = 10\text{–}20$ cm^{-1}, we have $\Delta\omega = 0.075\text{–}0.6$ cm^{-1}. Equation (10) gives the maximum intensity $I_{max} = I(t = +\infty) = (5\text{–}1.2)\times 10^9$ W/cm^2 and the intensity at the center of the "burnup" front $I(0) = I_{max}/2 = (2.5\text{–}0.6)\times 10^9$ W/cm^2.

2. SPATIAL STABILITY

Until now we considered solutions in the form of a plane wave $\mathscr{E}(t, z)$ neglecting the dependence on the transverse coordinates $r_\perp(x, y)$. It is known however that this dependence may result in instability of the solutions in the presence of small transverse perturbations,[10] which are manifested experimentally as a breakup of a beam with a smooth transverse intensity distribution into a system of filaments in which the energy density exceeds the threshold value for optical breakdown.[1] We shall analyze the stability of the solutions (2) and (10) by the method of Ref. 10 on the assumption that the complete solutions of Eqs. (1) and (8) describing quasiresonant amplification of a pulse $\mathscr{E}(z,t,\mathbf{r}_\perp)$ under diffraction and self-focusing conditions are

$$\mathscr{E} = \mathscr{E}_c(z, t) + \mathscr{E}_t(z, t, \mathbf{r}_\perp), \quad \mathscr{E}_c \gg \mathscr{E}_t, \tag{11}$$

where $\mathscr{E}_c$ is any of the solutions (3) and (10); $\mathscr{E}_t$ is a small transverse perturbation. Substituting Eq. (11) into Eq. (1) or Eq. (8), using the compensation condition (2) or (3), and retaining only the terms which are linear in respect of $\mathscr{E}_t$, we obtain

$$\begin{aligned}\frac{\partial \mathscr{E}_t}{\partial z} + \frac{i\Delta_\perp \mathscr{E}_t}{2k} = {}& \frac{\sigma N(z, t=0)}{2}\exp\left(-2\sigma\int_{-\infty}^{t} I_c\, d\theta\right)\\ & \times\left(\mathscr{E} - 2\sigma\,\mathscr{E}_c\int_{-\infty}^{t}(\mathscr{E}_c^*\mathscr{E}_t + \mathscr{E}_c\mathscr{E}_t^*)\, d\theta\right)\\ & \pm i\,\frac{\sigma N(z, t=0)\,\Delta\omega\, T_2}{2}\exp\left(-2\sigma\int_{-\infty}^{t} I_c\, d\theta\right)\\ & \times 2\sigma\,\mathscr{E}_c\left(\int_{-\infty}^{t}(\mathscr{E}_c^*\mathscr{E}_t + \mathscr{E}_c\mathscr{E}_t^*)\, d\theta\right) + ikn_2(\mathscr{E}_c^*\mathscr{E}_t + \mathscr{E}_c\mathscr{E}_t^*)\mathscr{E}_c,\end{aligned} \tag{12}$$

where $I_c = |\mathscr{E}_c|^2$. In this equation the plus sign in front of the third term corresponds to the amplification case of Eq. (3) and the minus sign corresponds to the case described by Eq. (10). Clearly, the latter case is preferred because it ensures mutual suppression of small-scale transverse perturbations [due to the last two terms on the right-hand side of Eq. (12)].

We shall show that this amplification case is more stable than the traditional one characterized by $\Delta\omega = 0$. Since Eq. (12) is linear with respect to $\mathscr{E}_t$, it is convenient to expand it as a Fourier integral[10]

$$\mathscr{E}_t = \iint \exp(i\varkappa\cdot\mathbf{r}_\perp)\,e(z, t, \varkappa)\,d^2\varkappa$$

and to write it down as a system for the real and imaginary parts $e = e_1 + ie_2$:

$$\frac{\partial e_1}{dz} - \frac{\varkappa^2}{2k}e_2 = \frac{\sigma N_0}{2}\exp\left(-2\sigma\int_{-\infty}^{t} I_c\, d\theta\right)\left(e_1 - \mathscr{E}_c 2\sigma\int_{-\infty}^{t} 2e_1\mathscr{E}_c\, d\theta\right); \tag{13a}$$

$$\frac{\partial e_2}{dz} + \frac{\varkappa^2}{2k}e_1 = \frac{\sigma N_0}{2}\exp\left(-2\sigma\int_{-\infty}^{t} I_c\, d\theta\right)\left(e_2 \pm \Delta\omega\, T_2\mathscr{E}_c 2\sigma\int_{-\infty}^{t} 2e_1\mathscr{E}_c\, d\theta\right) + kn_2 I_c 2e_1. \tag{13b}$$

In the absence of amplification ($N_0 = 0$) this system reduces to the Bespalov–Talanov system, from which the following dependence of the increment h of the growth of perturbations $e_{1,2} \approx \exp(hz)$ on the transverse wave number $\varkappa$ was obtained in Ref. 10:

$$h^2 = \varkappa^2(2n_2 k I_c - \varkappa^2/2k)/2k. \tag{14}$$

Our analysis will now be based on the following assumption: since the transverse perturbations usually appear because of the scattering of a wave being amplified by the boundary surfaces, microinclusions, and other defects of the active medium,[1] the evolution of these perturbations with time is (at least immediately after the scattering) practically the same as of the amplified wave $\mathscr{E}_c(z,t)$. This makes it

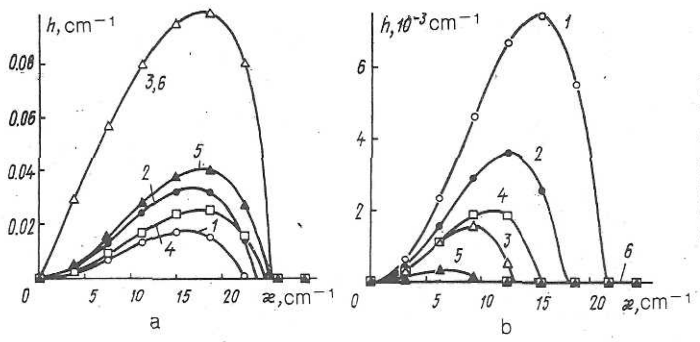

FIG. 5. Dependences of the local growth increments h of small-scale perturbations on the transverse wave number $\varkappa$ obtained for $\sigma N_0 = 1\ \mathrm{cm}^{-1}$ and $\Delta\Omega T_2 = 0.1$ under amplification conditions described by Eq. (3) (a) and Eq. (10) (b) in the case when $\Delta\omega = 0$ (1–3) corresponding to purely nonresonant self-focusing and in the case when $\Delta\omega \neq 0$ (4–6) corresponding to weak ($t \simeq 0$, curves 3 and 6), moderate ($t \gtrsim 0$, curves 2 and 5), and strong ($t \gg 0$, curves 1 and 4) saturations.

possible to simplify the analysis by rewriting $e_{1,2}$ in the form

$$e_{1,2}=\varepsilon_{1,2}(z,\ t,\ \varkappa)\,\mathscr{E}_c(z,\ t), \tag{15}$$

reducing the problem to the solution of two linear equations with z-dependent coefficients, where t occurs as a parameter:

$$\frac{\partial\varepsilon_1}{\partial z}-\frac{\varkappa^2}{2k}\,\varepsilon_2=-\varepsilon_1\,2\sigma N_0 g\,(z,\ t), \tag{16a}$$

$$\frac{\partial\varepsilon_2}{\partial z}+\frac{\varkappa^2}{2k}\,\varepsilon_1=\varepsilon_1 k n_2 I_c\,(z=0,\ t=0)\,\tilde{g}\,(z,\ t)$$

$$\pm\,\varepsilon_1\,2\sigma N_0\,\Delta\omega\,T_2 g\,(z,\ t), \tag{16b}$$

and in the case described by Eq. (3) we have

$$g=\frac{\ln\,(1+at-\sigma N_0 z)}{1+at-\sigma N_0 z},\ \tilde{g}=(1+at-\sigma N_0 z)^{-1},$$

whereas in the case described by Eq. (10), we obtain

$$g=\frac{\ln\,[1+\exp\,(\sigma N_0 z+at)]}{[1+\exp\,(\sigma N_0 z+at)]},\ \tilde{g}=[1+\exp\,(-\sigma N_0 z+at)]^{-1}.$$

It should be pointed out that the linear gain $e_{1,2}$ drops out from the above equations because of Eq. (15).

The behavior of the solutions of Eq. (16) as a function of the wave number can be analyzed conveniently by introducing a local increment $h(z,t)$ (for a given moment t and at a given point in the amplifier z). This makes it possible to consider the system (16) as one with constant coefficients at each point z, to obtain simple expressions for the local increment $h(z,t)$, and to compare their spatial stability by considering these expressions for different compensation conditions.

Figure 5a shows the dependences of the local increment $h(z,t)$ on the transverse wave number for the case described by Eq. (3), and we can see how the downward frequency detuning and saturation of the resonant transition increase the limiting wave number $\varkappa_{\lim}$ and the absolute value of the increment; perturbations of the resonant and nonresonant refractive index amplify one another. The physical meaning of this behavior is that the detuning $\Delta\omega$ downward from the transition frequency leads to self-focusing,[3] since in this case the resonant refractive index increases in the regions with a higher energy density and, consequently, with a stronger saturation (Fig. 1). Therefore, the compensation of the nonresonant refractive index in the case described by Eq. (3), i.e., primarily because of the static refractive index $\sigma N_0 \Delta\omega T_2$, does not ensure spatial stability, because the kinetics of the amplification is such that the self-focusing process is accelerated.

Figure 5b shows the dependences of the local increment $h(z,t)$ at the entry to an amplifying medium in the case described by Eq. (10) on the transverse wave number at different moments in time [in view of the fact that Eq. (10) is self-modeling, the results obtained at $z=0$ are valid for any value of z]. The maximum increments now correspond to the resonant refractive index, i.e., to the exact resonance $\Delta\omega=0$ [which is a situation opposite to the case described by Eq. (3)]. We can see that the increment $h_{\max}$ and the limiting $\varkappa_{\lim}$ decrease considerably in the case of weak saturation ($0 \gtrsim t > -\infty$), i.e., in the "burnup" front and ahead of it where fluctuations of the nonresonant refractive index due to fluctuations of $I(z,\ t)$ are manifested particularly clearly because of the resonant refractive index. On the other hand, in the case of strong saturation the values of $h_{\max}$ and $\varkappa_{\lim}$ increase to their values corresponding to $\Delta\omega=0$ since the resonant refractive index is suppressed and it cannot follow the nonresonant refractive index.

CONCLUSIONS

An analysis of the increments of small-scale perturbations shows that the positive nonlinearity of the refractive index of the insulator base of an active medium can be compensated using the variable part of the resonant refractive index due to saturation. This is ensured by detuning of the pulse frequency upward from a resonance and by profiling its time envelope in accordance with Eq. (10). An interesting feature of such a profile, which makes it easier to realize it in practice, is its identity with the shape of a steady-state bleaching wave in a saturable absorber.[7] This makes it possible to use in practice the mutual compensation as a complement to the familiar methods for suppressing self-focusing such as spatial filtering, amplification in diverging media, splitting of the medium into separate sections, etc.[1]

In practical applications of the mutual compensation in media characterized by an inhomogeneous broadening (for example, laser glasses) we need very detailed information on the resonant refractive index and its kinetics in the course of saturation, because such data determine the required frequency detuning and pulse profile. The optimistic estimates obtained above for a YAG:Nd^{3+} crystal are based on its well-known stimulated-emission cross section σ, inversion N_0, and law governing the variation of the resonant refractive index. Therefore, although the qualitative features of the mutual compensation are clearly retained also by a medium with an inhomogeneous broadening of the resonant transition, a quantitative description of the pulse profile and frequency detuning cannot be provided without allowance for the contribution of all the components of the spectral line and for their relative changes in the course of saturation.

It is interesting to note in conclusion that generation of