

Phase-conjugation of the isolated optical vortex using a flat surfaces.

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The robust method for obtaining the helical interference pattern due to the phase-conjugation of an isolated optical vortex by means of the non-holographic technique is proposed. It is shown that a perfect wavefront-reversal of the vortex in a linear polarization state via even number of reflections is achievable due to the turn of the photon's momentum $\vec{p} \approx \hbar \vec{k}$ with respect to the photon's orbital angular momentum projection L_z . The possible experimental realization is based upon *catseye - prism* like reflections inside the confocal optical *loop* cavity. The alternative scheme contains the Dove prism embedded in the optical *loop* with the odd number of reflections from mirrors. This *confocal* interferometric technique is applicable to the optical tweezers, atomic traps, Sagnac laser loops and metamaterials fabrication. © 2010 Optical Society of America

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1. Introduction.

Phase-conjugation (PC) proved to be an efficient tool for the laser beam divergence control [1], self-adjustment of optical schemes [2] and beam combination [3] a decades ago. A substantial progress in understanding of the physical mechanism of a PC-mirror is associated with a concept of the phase-singularities inside an optical speckle patterns [4]. In accordance with this concept the randomly spaced dark lines of the speckle (zeros of electric field complex amplitude $\mathbf{E}_f(z, \vec{r}_\perp, t)$) are collocated with the helical phase ramps [5]. Thus the phase-conjugated replica $\mathbf{E}_b(z, \vec{r}_\perp, t) = \mathbf{E}_f^*(z, \vec{r}_\perp, t)$ ought to have the set of the own helical phase ramps collocated with the phase ramps of the incident wave [2]. This helical phase feature of the optical speckle imposes a serious limitation upon the usage of the deformable adaptive mirrors because the smooth deformable surface is not capable to follow the helical phase ramp. On the other hand the dynamical interference pattern written by the incident speckle and reflected wave inside nonlinear optical medium, say Stimulated Brillouin Scattering (SBS) medium [3] or photorefractive medium [6], operates like a high-fidelity spatial filter increasing the signal-noise ratio for the backward reflected PC wave \mathbf{E}_b .

Recently the concept of phase singularity had been enriched by understanding that helical phase ramps are the sources of the helical interference patterns around zeros of the speckle optical fields [7, 8]. In particular it was shown that interference of the two counter propagating isolated optical vortices in the form of Laguerre-Gaussian (LG) beams produces a helical optical potential or "lattice with twist" [9]. The key point for achieving such a helical interference pattern proved to be the conservation of the *total* orbital angular momentum (OAM) in a PC-mirror: the turn of OAM of reflected wave is the urgent requirement to the perfect coincidence of the incident

and reflected wavefronts and helicoidal interference [8]. Noteworthy that for the non PC mirror the OAM is not reversed and the interference pattern around phase singularity is a toroidal one [10]. The other important feature of the PC-mirror is that OAM conservation leads unavoidably to the transfer of rotations to the PC mirror. In SBS mirror the rotations appear in the form of the helical acoustical phonons with $2\hbar$ OAM hence optical anisotropy (chirality) emerges in initially isotropic SBS medium [11]. Quite recently the chiral sound excitations in an initially isotropic liquid were found experimentally and obtained numerically using Khokhlov-Zabolotskaya-Kuznetsov equation [12]. Nevertheless we will show below that in a definite experimental conditions the PC reflection of a single optical vortex with a topological charge ℓ may be achieved experimentally with the even number of reflections from the perfectly flat (nonchiral) surfaces.

2. Propagation of the speckle and isolated vortex line.

The propagation of a speckle field along z -axis means a motion of the field zeros, i.e. the motion of the phase singularities in the same z -axis direction. The trajectories of zeros are not rectilinear [11, 13, 14], moreover trajectories intertwine each other as it happens with the higher-order LG optical vortices propagation [15]. The intertwining produces the structurally stable twisted entities in a speckle (fig.1) as is shown by numerical modeling of the following equation [7]:

$$\frac{\partial \mathbf{E}_{(f,b)}(z, \vec{r}_\perp, t)}{\partial z} + \frac{n(z, \vec{r}_\perp)}{c} \frac{\partial \mathbf{E}_{(f,b)}}{\partial t} \pm \frac{i}{2k_{(f,b)}} \Delta_\perp \mathbf{E}_{(f,b)} = 0, \quad (1)$$

where $n(z, \vec{r}_\perp)$ is inhomogeneity of refractive index, $k_{(f,b)} = |\vec{k}_{f,b}| \approx k_z$ are the wave numbers of the counter directed incident and reflected speckle fields,

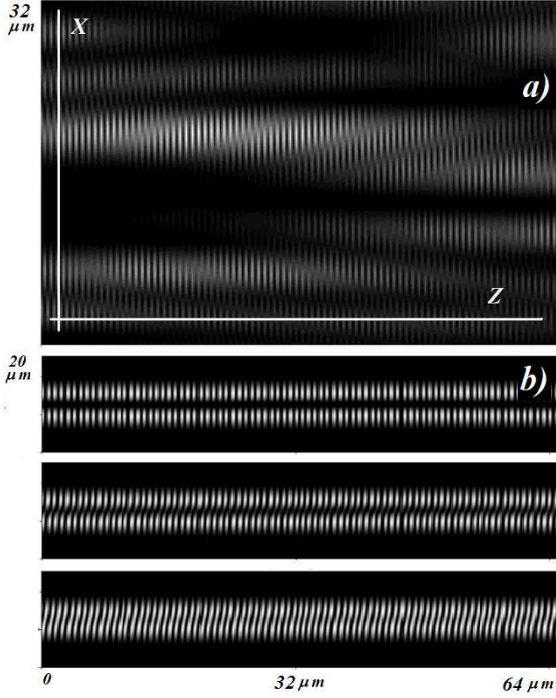


Fig. 1. a) Intertwining of the helical interference patterns in the optical speckle [7]. The size of pattern is $32\mu\text{m}$ in X-direction and $64\mu\text{m}$ in Z-direction. The period of longitudinal (Z) modulation is $\lambda/2$. b) In contrast to optical vortices in a speckle the isolated LG optical vortices propagate rectilinearly. Interference pattern (4) is sliced at $Y = 0, 5, 10\mu\text{m}$ distances from the vortex axis Z.

with boundary condition as a multimode random field [16] composed of N_g plane waves having amplitudes A_{j_x, j_y} , random phases θ_{j_x, j_y} and randomly tilted wave vectors each having random transverse projections \vec{k}_{j_x, j_y}^\perp at $z = 0$ plane:

$$\mathbf{E}_{(f)}(\vec{r}, 0) \approx \mathbf{E}^0_{(f)} \sum_{j_x, j_y \in N_g} A_{j_x, j_y} \exp [i \vec{k}_{j_x, j_y}^\perp \cdot \vec{r}_\perp + i \theta_{j_x, j_y}]. \quad (2)$$

The paraxial propagation of the randomly tilted plane waves produces the twisted interference patterns resembling visually the *ropes* each composed of several intertwined optical vortices [7, 15]. The similar propagation behavior and appearance of the *knot* structures had been reported in [13, 14].

In contrast to the speckle field, the isolated vortex line propagates rectilinearly in a free space and it is structurally stable (fig.1). This happens for example for the LG laser beam with topological charge ℓ [18]:

$$\mathbf{E}_{(f,b)}(z, r, \theta, t) \sim \frac{\mathbf{E}^0_{(f,b)} \exp [i(-\omega_{(f,b)}t \pm k_{(f,b)}z) \pm i\ell\theta]}{(1 + iz/(k_{(f,b)}D_0^2))^2} (r/D_0)^\ell \exp \left[-\frac{r^2}{D_0^2(1 + iz/(k_{(f,b)}D_0^2))} \right], \quad (3)$$

where $[(z, r, \theta, t) \rightarrow (z, \vec{r}_\perp, t)]$ are cylindrical coordinates embedded at LG axis ($z - axis$). This straight vortex

line is the exact self-similar solution of the free space wave equation in a paraxial approximation (1). Our aim is to describe how to use the structural stability, hence self-similar propagation of the optical vortex for the non-holographic wavefront reversal by conventional mirrors, lenses and prisms. At the first sight our proposal looks a counterintuitive one, because we focus attention of experimentalists upon previously criticized *catseye* prism PC techniques [2]. The case is that the *rays reversal* (with small lateral displacement) inside the prism is not able to perform a *wave propagation* reversal of the random collection of optical vortices in a speckle field or in the complicated image optical field. This seeming paradox is resolved by taking into account that rays reversal means a photon's *momentum* reversal, while helical phase singularity is reversed by means of the *angular momentum* reversal [8]. Noteworthy that OAM direction is changed to the opposite one inside Dove prism due to one total internal reflection inside prism (at 45 degrees incidence angle) and two refractions [19]. The same happens in *catseye* prism due to the same reason: when the plane surface is tilted at the angle α with respect to propagation of vortex the rotational symmetry of setup is absent hence the angular momentum is not conserved and OAM is rotated at 2α angle. For both prisms the change of angular momentum is $2\hbar$ per photon hence the prisms feel the torque $|\vec{T}| = 2 \cdot I/\omega$, where I is intensity, ω is radiation frequency [8]. As a consequence of OAM reversal the vortex propagates in the optical *loop* schemes (fig.2, fig.3) as a perfectly phase-conjugated one due to the simple reflections from conventional prism surfaces, provided the vortex is slightly focused by a thin lenses in order to compensate diffractive divergence. The technical requirements for the *loop* adjustment are the same as those previously formulated for the ring lasers and Fabry-Perot cavities with Hermite-Gaussian and Laguerre-Gaussian beams [10].

3. Angular momenta orientation and rotation of interference pattern

For the ultimate quality PC reflection of the linearly polarized ($\mathbf{E}^0_{(f,b)} || \mathbf{y} - axis$) ℓ 's order LG-laser beam the interference pattern inside the beam waist reads as:

$$|\mathbf{E}_f(z, r, \theta, t) + \mathbf{E}_b(z, r, \theta, t)|^2 \sim |\mathbf{E}^0_{(f,b)}|^2 \cdot (r/D_0)^{2\ell} \exp \left[-\frac{2 \cdot r^2}{D_0^2(1 + iz/(k_{(f,b)}D_0^2))} \right] \cdot [1 + \cos[(\omega_f - \omega_b)t - (k_f + k_b)z + 2\ell\theta]], \quad (4)$$

The helicity of pattern is due to the self-similar phase argument $(\omega_f - \omega_b)t - (k_f + k_b)z + 2\ell\theta$ which remains a constant at the double helix with a diameter $\sim 2D_0$ and $\lambda/2$ pitch ($\lambda = 2\pi/k_{(f,b)}$) [8]. Such double helix optical potential rotates with angular frequency $\Omega = \omega_f - \omega_b$ which looks attractive from the point of view of optical microfluidics, micro and nano-particles manipulation [6, 15] and as an optical dipole trap for ultracold atomic ensemble [9].

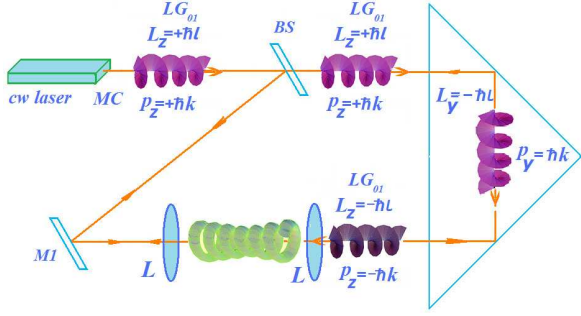


Fig. 2. Two consecutive reflections of LG beam emitted by CW laser with mode converter MC. Each total internal reflection inside the *catseye* prism from plane prism surface changes the topological charge of LG ℓ to the opposite one. After two reflections at 45-degree angle the optical vortex has the opposite direction of propagation and opposite direction of the angular momentum. The counter propagating LG has the same topological charge ℓ hence composite wavetrain produces the helical interference pattern. Confocal telescope consisting of the thin lenses L compensates the free-space propagation parabolic wavefront.

The key point in physical interpretation of this helical pattern is the mutual orientation of the photons momentum $\vec{p} \approx \hbar\vec{k}$ and projection of the photon's orbital angular momentum L_z on propagation axis [8, 18]. The mutual orientation of both quantum and classical momenta \vec{p} and angular momenta \vec{L} is changed after single reflection from isotropic optical element namely metal or multilayer dielectric mirror. On the contrary the anisotropic structures inside wavefront reversal mirror [8] perform turn of the orbital angular momentum of laser beam because of the wavefront matching property of PCM. This turn operation is analogous to the photon's spin turn (change of the circular polarization to the oppositely rotating one) when passing through birefringent plate (i.e. anisotropic crystal) [20].

Consider two optical *loop* schemes (fig.2, fig.3) composed of plane mirrors, ideal thin lenses for the adjusting of the parabolic component of the wavefronts (3) [19] and prisms (possibly with laser gain medium inside). As is shown in [8] each reflection from plane mirror changes the mutual orientation of the photons momentum $\hbar\vec{k}_z$ and the angular momentum $L_z = \pm\hbar\ell\vec{z}/z$ to the opposite one.

Consequently two reflections in (fig.2) scheme does not change the topological charge of photon and oppositely propagating wave possesses the helical wavefront with the same handedness. Thus LG beam reflected inside *catseye* prism and the other LG beam reflected from beamsplitter BS and mirror M1 will have the perfect wavefront coincidence provided their parabolic phase profiles which occurs due to a free-space propagation are compensated by a thin lens (fig.2, fig.3). As a result the interference pattern will have a double helix geometry,

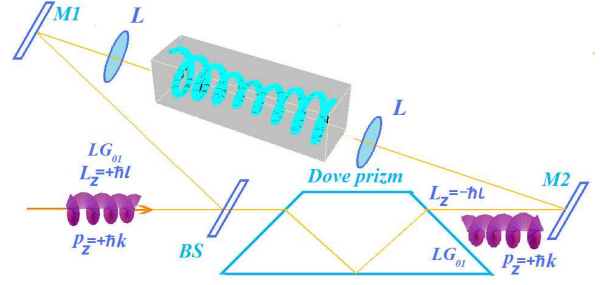


Fig. 3. The single reflection inside Dove prism changes the topological charge. After the two refractions the LG propagates with conserved momentum and overturned angular momentum. This leads to helical interference pattern due to overlapping with counter propagating LG, having opposite OAM.

provided their path difference is smaller than coherence a length $\Delta l_{coh} = c \cdot \tau$ (τ is coherence time). Alternatively in fig.3 scheme the *single* reflection inside Dove prism changes the topological charge of each photon to the opposite one [19] and the else reflection from mirror M2 is needed to restore the mutual orientation of the OAM and momentum. This sequence of reflections ensure the helical wavefront coincidence and produces the helical interference pattern with the twice-reflected (BS+M1) counter propagating LG beam. The removal of Dove prism will produce toroidal interference pattern because of the absence of phase conjugation and parallel orbital angular momenta of colliding photons [6, 8, 10].

The frequency shift Ω may be produced via two different mechanisms. The first mechanism is the rotational Doppler shift which arises because of rotation of the birefringent half-wavelength plate which alternates the spin component of angular momentum [15, 21] or rotating Dove prism, which alternates the orbital component of angular momentum [22]. The Dove prism rotation technique is difficult to implement because of strict alignment requirements for interference pattern control. The other mechanism is the Sagnac frequency shift which appears in a ring laser located in rotating reference frame. This happens when prisms have laser gain areas collocated with LG beam propagation. Typically the optical gain is induced in a rare-earth doped dielectric host crystals by virtue of the diode laser pump [23]. In this case the external laser outside the *loop* is not necessary and the beamsplitter BS is to be replaced to return mirror R3. The conditions for the selection of a given transverse LG mode are to be fulfilled [24] and such a case deserves a special consideration elsewhere. As is well known for the *loop* laser schemes the counter propagating beams have a different frequencies ω_f and ω_b because of the Earth rotation having angular frequency Ω_{\oplus} and the angular frequency of the optical table rotation Ω_{iab} . For the such *Sagnac loop* [24] the frequency splitting is:

$$\Omega = (\omega_f - \omega_b) = \frac{16\pi^2 A \Omega_r}{P \cdot \lambda}, \quad (5)$$

where $\Omega_r = \Omega_{\oplus} + \Omega_{lab} \sim (2\pi/86400) + \Omega_{lab}$ is the angular speed of rotation of the laboratory frame, P , A are the perimeter and the square of the loop respectively. The frequency shift is measured by a detection of a beats (rotation of interference pattern in our case) of the counter propagating intracavity beams behind the cavity mirrors (M1, M2 in fig.2, fig.3). For the typical ratio of the spatial dimensions of the *Sagnac loop laser* to the wavelength $\lambda \sim 1\mu m$ the frequency splitting proves to be $\Omega \approx 2\pi 10^{-(1-3)} rad/sec$. In particular the evaluation of Ω is straightforward for the circular ring cavity of radius R when $P = 2 \cdot \pi R$, $A = \pi R^2$: the frequency splitting is $\Omega = \Omega_r 8\pi^2 R/\lambda$.

4. Conclusion

In summary we proposed the phase-conjugation of an isolated optical vortex line (LG-beam) with lateral displacement in the confocal optical *loop* scheme with the *even* number of reflections. The alternative optical loop with the *odd* number of mirrors contains a Dove prism which alternates the photon's OAM projection after the straight passage through a prism. This scheme is different from Mach-Zehnder setup used previously for rotational Doppler effect study [15, 21, 22]. Our loop setups with colliding phase-conjugated optical vortices and helical interference patterns therein are the promising tools for nonexpensive replacement of nonlinear optical phase conjugators based upon SBS [2, 3], photorefractive crystals [6] and liquid crystal light valves. The field of experimental applications of confocal loops with catseye prism or Dove prism is in atomic traps [9] and optical tweezers, in particular in assembling the protein - like clusters [25]. The other intriguing application is in the lithography of metamaterials [26, 27] and optical waveguides with the helical refractive index and conductivity [28].

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