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Letter

The effect of roughness of optical elements on the transverse structure of a light field in a nonlinear Talbot cavity

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Abstract. It is shown numerically that in a Talbot cavity being formed by a thin nonlinear amplifying layer and a pair of a flat imperfect mirrors, the small phase distortions place a fundamental limitation on the size of the aperture with in-phase lasing. This effect results in a spreading of the far field distribution and it is not suppressed by plane-wave injection.

The phenomenon of self-imaging leads to the formation of periodic transverse structures in optical resonators [1]. In contrast to Gaussian beams the conditions for the formation of such structures restrict the resonator length to a multiple of p^2/λ, where p is the period of the structure and λ is the wavelength. A failure of this condition leads to distortions of the periodic modes. For example, a Talbot cavity designed for the synchronization of a set of lasers [2, 3] can synchronize only a finite number of lasers because of a small spread in wavelengths from one laser to another [4]. Similar distortions should arise also in a Talbot cavity proposed for diode array end-pumped solid state lasers [5]. This cavity is formed by a flat mirror and a thin nonlinear amplifying Nd layer. The latter is pumped by a diode array through the back surface that is transparent for the pump and totally reflecting for the lasing wavelength (see figure 2 [5]). As a result a spatially periodic grid of inversion is formed selecting transversely periodic eigenmodes under the Talbot conditions [5]. The surfaces of the elements of the cavity usually have a roughness of the order of $\lambda/10$ or less. So there is a slightly chaotic variation in the length of the resonator from one point to another in the x–y plane. Our goal here is to study the influence of these phase distortions on the wavefront of the laser output.

The numerical model [5] that describes the formation of the transverse modes computes the one-dimensional transverse structure $E_n(x, z)$ from n to $n+1$ pass:

$$E_{n+1}(x,z)=\left(\frac{ik}{2\pi z}\right)^{1/2}\exp(ikz)\int_{-\infty}^{+\infty}\exp\left[\frac{-ik(x-x')^2}{2z}\right] \times D(x')f[N(x'),E_n(x',z=0)]\,dx', \qquad (1)$$

where $z=2p^2/\lambda$ (round-trip length of resonator), $N(x)$ is inversion, $f(N, E_n)$ is the saturated gain [5] (taken equal 3 for a small signal). $D(x)$ is the complex function describing the nonresonant losses of the cavity whose amplitude is a Fermi–Dirac function

$$R^{1/2}\Big/\left[\exp\left(\frac{|x|-x_d}{T}\right)+1\right],$$

where $R=0{\cdot}95$ is the reflectance of the output mirror, the 'Fermi energy' x_d defines the edges of the diaphragm, the nonzero 'temperature' T smooths the edges approximating the real multilayer dielectric mirrors (this kind of 'soft' diaphragm also reduces the requirements to FFT numerical procedure). The phase of D has the form:

$$\arg D(x)=\left|\sum_{m=1}^{N_h}\exp\left\{\mathrm{i}\left[\left(\frac{mx}{N_p p}\right)+\psi_m\right]\right\}\right|^2, \tag{2}$$

where ψ_m is a random number from the interval $[0,\pi]$, N_h and N_p are correspondingly the number of harmonics used to model the random function and the number of periods of the amplifying grid (this form of 'noise' function is also used to model partially coherent signal or random walks, see for example [6, 7]). For $N_h \geqslant 100$, the random function $\arg D(x)$ tends to be δ-correlated, remaining of course continuous and differentiable. Its mean value $\langle \arg D(x)\rangle$ was considered to be a measure of roughness and was normalized to cover the surface ripple in the range $0{\cdot}01\lambda$–$0{\cdot}1\lambda$. Of course the above model is in some sense qualitative because the real spectrum of the intracavity phase distortions is unknown *a priori*. Moreover, the real intracavity distortions need not be purely chaotic for all kinds of optical surfaces. Therefore, the numerical results listed below should be considered as giving only qualitative estimates. Strictly speaking, to obtain a quantitative agreement with a particular experimental situation it is necessary to adjust the amplitudes and phases of the ripple harmonics in equation (2).

The integral in equation (1) was evaluated by an efficient FFT procedure using arrays consisting of 2^{11} points for $N_p=16$ and 2^{13} points for $N_p=64$. In all runs the initial condition ($n=0$) for $E_n(x, z)$ was chosen as a weak plane wave which provided the selection of the in-phase mode [1]. The convergence was checked by observing the power

$$\int |E_n(x)|^2\,\mathrm{d}x,$$

and the autocorrelation trace

$$\frac{\left|\int E_n(x)E^*_{n+1}(x)\,\mathrm{d}x\right|^2}{\int |E_{n+1}(x)|^2\,\mathrm{d}x\int |E_n(x)|^2\,\mathrm{d}x}.$$

The power saturated within a 1% accuracy after 32 and 35 iterations for $N_p=16$ and $N_p=64$, respectively, while autocorrelation exceeded $0{\cdot}99$ after 5 iterations in both cases, because of the plane wave initial condition. The results of computations for $N_p=64$ (see figure 1) showed that single lobe far field pattern is achievable only for a sufficiently small mean amplitude of roughness $\langle \arg D(x)\rangle$ (namely $0{\cdot}025\lambda$ for $N_p=64$). For a larger value of the roughness, side bands arise in the far field and reduce the brightness. These side bands are produced by those parts of the near field which have a tilted wavefront (see figure 1). Note that equiphasing of the wavefront after the injection of the plane wave, reported in [4], was not observed in the above mentioned runs. For a larger number of periods of the amplifying grid N_p the effect of roughness is stronger. On the other hand for $N_p=16$ the single lobe far field output is stable even for $\langle \arg D(x)\rangle=0{\cdot}1\lambda$ (see figure 2).

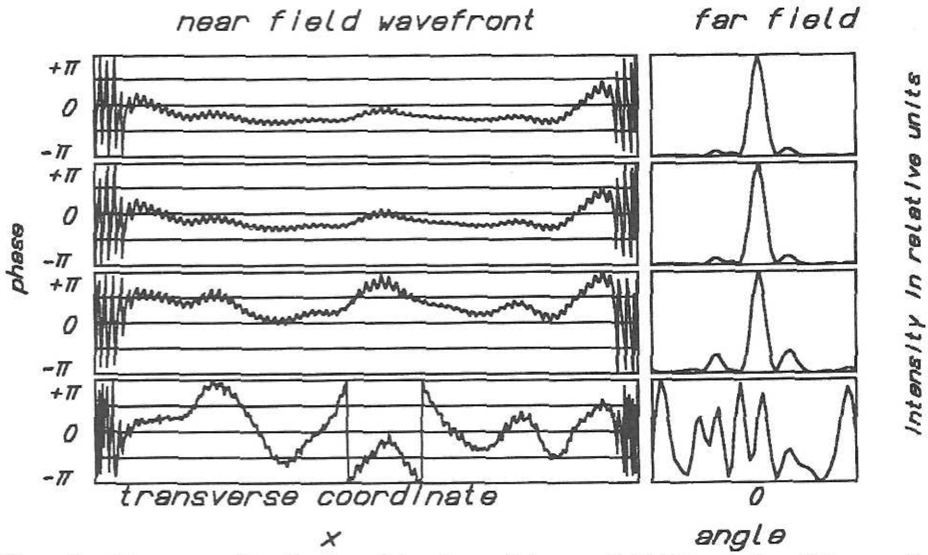

Figure 1. Transverse distribution of the phase of the near field $E_n(x, z)$ after 35 iterates (left column) and corresponding far field intensity distribution (right column). $\langle \arg D(x) \rangle$ gradually increases in value (0·001λ, 0·005λ, 0·01λ, 0·025λ) from the top region to the bottom one. The number of periods of the amplifying grid is $N_p = 64$.

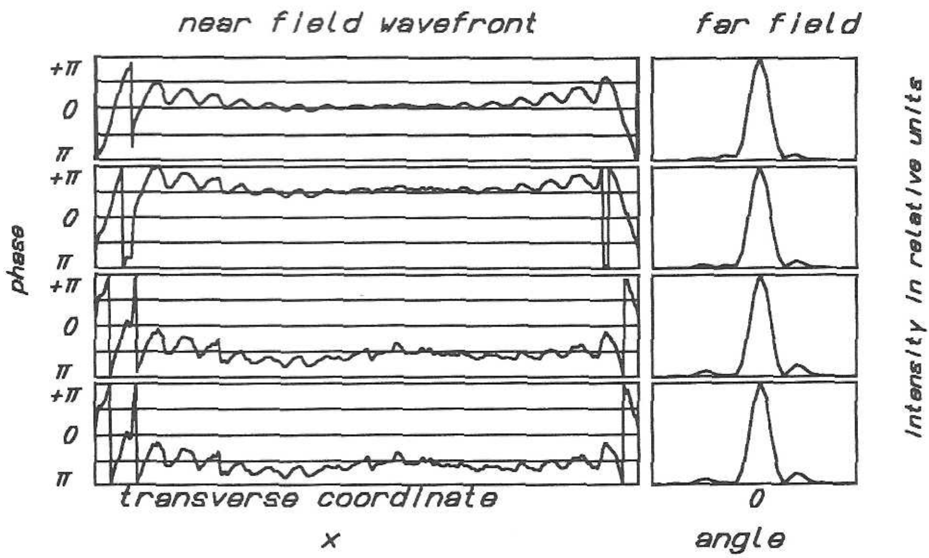

Figure 2. The same distributions as in figure 1 but the number of periods of amplifying grid is $N_p = 16$.

Thus we see that because of the presence of the low-amplitude chaotic phase mask which always exists even for the most perfect optical surfaces it is impossible to obtain a perfect in-phase lasing for large aperture Talbot cavities. This effect takes place even if the spread of the eigenfrequencies [4] is absent. The small but inevitable transverse phase modulation will also reduce the number of synchronized elements in self-imaging phase-locked diode arrays [2].

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