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Optical and Sound Helical Structures in a Mandelstam–Brillouin Mirror

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It is shown theoretically that the phase conjugation of a speckle optical field in a Mandelstam–Brillouin mirror is accompanied by the excitation of helical hypersonic waves with a step equal to one-half of the optical wavelength. The excitation of these waves violates the initial isotropy of the dielectric medium. The predicted effect admits clear physical interpretation based on the angular momentum conservation. The angular momentum transfer from the light to the medium occurs in the vicinity of an optical singularity (optical vortex line) due to reversal of the light orbital angular momentum by the phase-conjugation mirror. The excitation of hypersonic waves transferring the angular momentum is the necessary condition for the reversal of the angular momentum of the reflected light.

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The phenomenon of phase conjugation by a Mandelstam–Brillouin nonlinearity has been discovered for a speckle optical field [1, 2]. Optical radiation described by the field $\mathbf{E}_p(z, \mathbf{r}, t)$ propagated through a transparent dielectric plate with random phase irregularities whose spatial scales were slightly greater than the wavelength [3] excited the counterpropagating phase-conjugated Stokes wave in a lossless-dielectric multimode optical waveguide. It is known than the propagation of the phase-conjugated wave $\mathbf{E}_s = \mathbf{E}_p^*$ imitates the "time reversal" of the initial pump wave $\mathbf{E}_p$. Since the wavenumbers $k_p = \omega_p/c$ and $k_s = \omega_s/c$ of the initial and phase-conjugate wave, respectively, differ only by about 10^{-5} [4], the amplitudes and phases of $\mathbf{E}_p$ and $\mathbf{E}_s$ are identical in any plane perpendicular to the propagation direction (z axis).

A surprising feature of this phenomenon is a radical improvement of the phase-conjugation fidelity, i.e., a significant increase in the correlation coefficient $H(z)$ [1] between the incident $\mathbf{E}_p$ and reflected $\mathbf{E}_s$ waves, given by the formula

$$H(z) = \frac{\left|\int \mathbf{E}_p \mathbf{E}_s^* d^2\mathbf{r}\right|^2}{(\int |\mathbf{E}_p|^2 d^2\mathbf{r})(\int |\mathbf{E}_s|^2 d^2\mathbf{r})}, \quad (1)$$

if the incident laser beam passed through a random phase plate. On the contrary, the phase conjugation fidelity turned out to be much lower for beams with smooth transverse amplitude and phase distributions, e.g., zero-order Gaussian beams. In this case, a significant fraction of the reflected light scatters into the higher-order modes [3]. A similar decrease in the conjugated-wave relative fraction was observed in the case of the phase conjugation of isolated lower-order Gaussian beams with a phase singularity, i.e., the first-order Laguerre–Gaussian beams [5]. It is known that the phase singularity is a specific wavefront point at which the phase is indeterminate. The wavefront has the form of a spiral staircase in the proximity of such a singular point [6]. A Laguerre–Gaussian beam is described by the well-known elementary solution of the scalar wave equation in a linear medium for the fixed polarization state (e.g., linear or circular) [7, 8]. The form of this solution is identical to the eigenfunction of the first excited state of a two-dimensional quantum harmonic oscillator in an axially symmetric parabolic potential [9]. In the cylindrical coordinates (z, r, ϕ), where z is the propagation direction, we have

$$\begin{aligned} &\mathbf{E}_{(p,s)}(z, r, \phi, t) \\ &\approx \mathbf{E}^0_{(p,s)} \exp[i\omega_{(p,s)}t \mp (ik_{(p,s)}z + il\phi)] \\ &\times r^l \exp\left[-\frac{r^2}{(D^2(1 \pm iz/k_{(p,s)}D^2))}\right], \end{aligned} \quad (2)$$

where $\mathbf{E}^0_{(p,s)}$ are the amplitudes of the pump (p) and Stokes (s) scattered counterpropagating waves, $\omega_{(p,s)}$ and $k_{(p,s)}$ are their carrier frequencies and wavenumbers, respectively, $l = 0, \pm 1, \pm 2, \ldots$ is the azimuthal quantum number or Laguerre–Gaussian topological charge, and D is the beam diameter. The periodicity $\mathbf{E}$

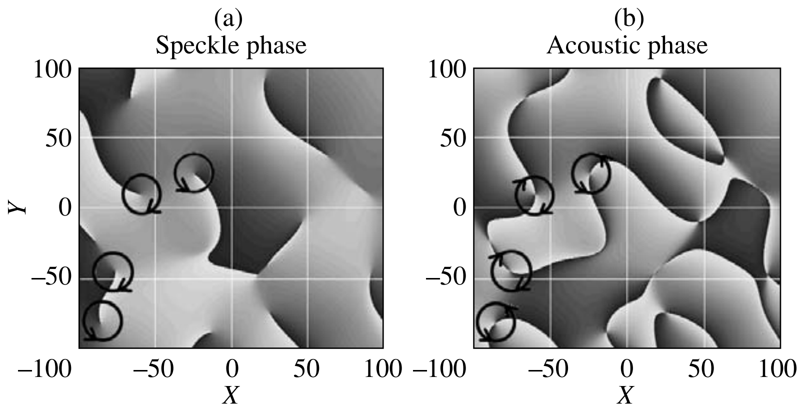

Fig. 1. Phase dislocations of (a) optical Arg[$\mathbf{E}_p(x, y, z)$] and (b) acoustic Arg[$Q(x, y, z)$] fields in the volume of a Mandelstam–Brillouin mirror. The grey scale corresponds to various values of the phase in the range from (black) 0 to (white) 2π. The grey circles mark two vortex–antivortex pairs, i.e., indeterminate-phase singular points. The single-arrowed circles (unit topological charge) correspond to the optical vortices. The acoustic-vortex topological charge is doubled (two-arrowed circle) because the orbital angular momentum is transferred to the medium. The phase changes by $\pm 2\pi$ and $\pm 4\pi$ along a closed path enclosing statistically typical (most frequently observed) optical and acoustic singularities, respectively. Optical- and acoustic-vortex locations are identical. The sizes along the x and y axes are 100 μm. The pictures correspond to the z planes in which the correlation coefficient $H(z)$ is close to unity. This is possible if z is small and the stimulated Mandelstam–Brillouin scattering mirror has a reflection coefficient of about 0.1 (weak saturation), i.e., near the output window of the stimulated Mandelstam–Brillouin scattering cell.

with respect to the azimuthal angle ϕ results in the quantization of the angular momentum transferred by optical photons [10].

It is shown in [8] that the inclusion of the counter-propagating-wave interference provides significantly deeper insight into the phase-conjugation physical mechanism. The interference pattern (the radiation intensity distribution) in the vicinity of a phase singularity is formed by the superposition of two counter-propagating waves and has the form of a right- or left-handed double helix for a wavefront with right- or left-handed dislocation, respectively:

$$\begin{aligned} \mathbf{I}_{\rm light} &\approx |\mathbf{E}(z, r, \phi, t)|^2 \\ &\approx |\mathbf{E}_{\rm p}(z, r, \phi, t) + \mathbf{E}_{\rm s}(z, r, \phi, t)|^2 \\ &\approx 2|\mathbf{E}^0_{\rm p,\,s}|^2[1 + \cos[(\omega_{\rm p} - \omega_{\rm s})t - (k_{\rm p} + k_{\rm s})z + 2l\phi]] \\ &\quad \times r^{2l}\exp\left[-\frac{2r^2}{D^2(1 + z^2/k_{\rm p}^2 D^4)}\right]. \end{aligned} \quad (3)$$

Equation (3) shows that the spatial profile of the helical structure is given by the self-similar variable $(\omega_p - \omega_s)t - (k_p + k_s)z + 2l\phi$, where the first two terms describe a moving Bragg lattice providing for the Doppler frequency shift $\omega_p - \omega_s = 2\omega_p v_{ac}/c$ of the reflected radiation [1, 2] (v_{ac} is the speed of sound). The third term $2l\phi$ appearing due to the phase conjugation is responsible for the helical-structure formation. The sign of the quantum number (topological charge) l determines the helix hand. The positive and negative l values correspond to the right- and left-handed helices, respectively. Moreover, owing to the difference between the pump- and conjugated-wave frequencies, the helical interference pattern is nonstationary and rotates around the phase dislocation with a frequency equal to the difference between the incident- and conjugated-wave frequencies, i.e., hypersonic frequency $\Omega_{ac} = \omega_p - \omega_s$. In the case of frequency down-conversion when phase conjugation starts from spontaneous noise, the interference pattern rotates clockwise around any vortex line. On the contrary, in the case of anti-Stokes frequency conversion by, e.g., reflection from a four-wave stimulated Mandelstam–Brillouin scattering mirror, the interference pattern rotates counterclockwise. The complex amplitude of the scalar hypersonic field Q_{ac} for the stationary stimulated Mandelstam–Brillouin scattering is given by the known formula [1, 8]

$$\begin{aligned} Q_{\rm ac} &\approx \mathbf{E}_{\rm p}\mathbf{E}_{\rm s}^* \\ &\approx \exp[+i[(\omega_{\rm p} - \omega_{\rm s})t - (k_{\rm p} + k_{\rm s})z + 2l\phi] \\ &\quad \times r^{2l}\exp\left[-\frac{2r^2}{D^2(1 + z^2/k_{\rm p}^2 D^4)}\right]. \end{aligned} \quad (4)$$

Using this formula, it is easy to find the hypersound amplitude and phase distributions in any z plane (Fig. 1) if the distributions of the pump wave $\mathbf{E}_p$ and the Stokes wave $\mathbf{E}_s$ are known. The Stokes wave is the phase-conjugated pump wave $\mathbf{E}_p^*$ in the case of perfect phase conjugation. Hence, according to Eq. (4), the amplitude and phase distributions of the hypersonic field Q_{ac} can be found by taking the square of the pump-field distribution. It is known from [11] that the numbers of right- and left-handed wavefront dislocations in a speckle field are equal with high accuracy. This corresponds to the current generally adopted view of turbulence as a chaotic ensemble of "vortex–antivortex" pairs (Fig. 1) connected by domain walls. According to the theory of electromagnetic-field helical dislocations [6], the turbulent-field vortex lines are not rectilinear, in contrast to the generation of spatially periodic vortex lattices with intermittent velocity-circulation values in an optical cavity [7, 12–14]. Numerical simulations show that the vortex lines of a speckle field form a set of interlaced "snaky" trajectories (specklons) [1]. Viewing the interference pattern along the z axis by distances exceeding the Fresnel length $L_R \approx k_p D^2$ reveals the continuous creation and annihilation of vortex–antivortex pairs. At the same time, the Cauchy problem for the parabolic equation describing the propagation of the pump wave $\mathbf{E}_p$ along the z axis under the initial condition in the form of a multimode random process at $z = 0$ (speckle field) can be solved analytically in the undepleted-pump approximation [15]:

$$\mathbf{E}_p(\mathbf{r}, z = 0) \approx \mathbf{E}_p^0 \sum_{0 < j_x, j_y < N_G} A_{j_x, j_y} \times \exp\left[i2\pi\left\{ \frac{x j_x}{p_x} + \frac{y j_y}{p_y} + i\theta_{j_x, j_y} \right\}\right]. \quad (5)$$

Here, θ_{j_x, j_y} are random numbers in the interval $[0, \pi]$, A_{j_x, j_y} are the real amplitudes, p_x and p_y are the maximum transverse sizes in the (x, y) plane, $\mathbf{r} = (x, y) = (r, \phi)$, and j_x and j_y are integers numbering random-phase plane waves. In this case, the amplitude and phase distribution of $\mathbf{E}_p$ for any z value has the form [12]

$$\mathbf{E}_p(\mathbf{r}, z > 0) \approx \mathbf{E}_p^0 \exp(ikz) \sum_{0 < j_x, j_y < N_G} A_{j_x, j_y} \times \exp\left[i2\pi\left\{ \frac{x j_x}{p_x} + \frac{y j_y}{p_y} + \frac{\pi z}{k_p}\left(\frac{j_x^2}{p_x^2} + \frac{j_y^2}{p_y^2} \right)\right\} \exp[i\theta_{j_x, j_y}]\right]. \quad (6)$$

The zero-amplitude singular points in the (x, y) plane are also the centers of helical phase dislocation [1, 6], as shown in Fig. 1a corresponding to Eq. (6) with the number of the spatial harmonics $N_G = 32$. The singular-point positions slowly evolve along the z axis, as seen in Fig. 2. Each vortex line corresponding to a phase dislocation bends slowly. As a rule, each dislocation is neighbored by another dislocation with the opposite topological charge [11]. In the context of work [8], this means that the total angular momentum of the vortex–antivortex pair is close to zero [7], while interference with the counterpropagating waves forms a set of randomly located helical interference patterns (Fig. 3) in the same way as in the cases of interference of a single optical vortex with the phase-conjugated counter-vortex [8] or in a dusty plasma [16].

According to Eq. (4), the hypersonic-field structure formed in the volume of the stimulated Mandelstam–Brillouin scattering mirror due to the interference of counterpropagating waves [1] correlates with the radiation-intensity distribution, i.e., has the form of a set of randomly spaced pairs of counter-handed helices (Fig. 3). In this case, despite the rotation of the pairs the acoustic-wave velocity field turns out to be continuous due to the presence of neighboring counter-rotating vortex pairs [7].

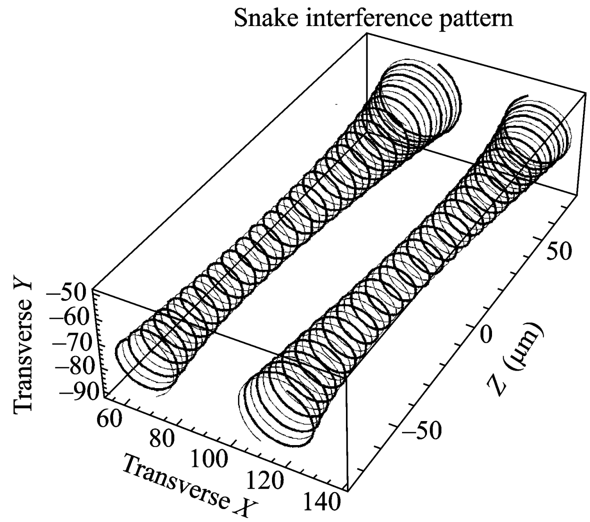

Fig. 2. Vortex–antivortex pair of an optical speckle field. The interference of counterpropagating phase-conjugated waves with phase singularities forms a pair of neighboring double opposite-handed helices displaced along the z axis by one-half of the wavelength $\lambda_{p,s} \approx 1$ μm. The typical transverse size is $\langle D \rangle \approx 20$–$50$ μm. The typical mean longitudinal size before the pair annihilation is $k_{p,s}\langle D \rangle^2 \approx 400$–$2500$ μm. Both double helices rotate in the same direction with the acoustic frequency $\Omega_{ac} = \omega_p - \omega_s$. The optical-helix rotation is synchronized and has the high spatial correlation with the hypersound helices. The rotation hand reverses upon the sign reversal of Ω_{ac}. The right- and left-handed helix turns move in the positive and negative z directions, respectively. As a result, the right- and left-handed helices act as "Archimedean screws" transferring the density perturbations in the opposite directions.

The above analysis implies that an acoustic speckle field formed by phase singularities also carries the angular momentum, although acoustic waves are scalar and would not have polarization. Indeed, the "spin component" of the acoustic-field angular momentum (Eq. (4)) is almost zero in this case. However, the orbital angular momentum is noticeable. Owing to the conservation of the angular momentum $L_z = \pm l\hbar$ carried by a pump photon, the Mandelstam–Brillouin scattering by an acoustic phonon is accompanied by the transfer of the double orbital angular momentum $L_{ac} = 2l\hbar$. As a result, the reflected phase-conjugated Stokes photon has the reversed orbital angular momentum $L_z = \mp l\hbar$ [8]. This is taken into account by doubling the argument of the phase factor $\exp[i2l\phi]$ (topological

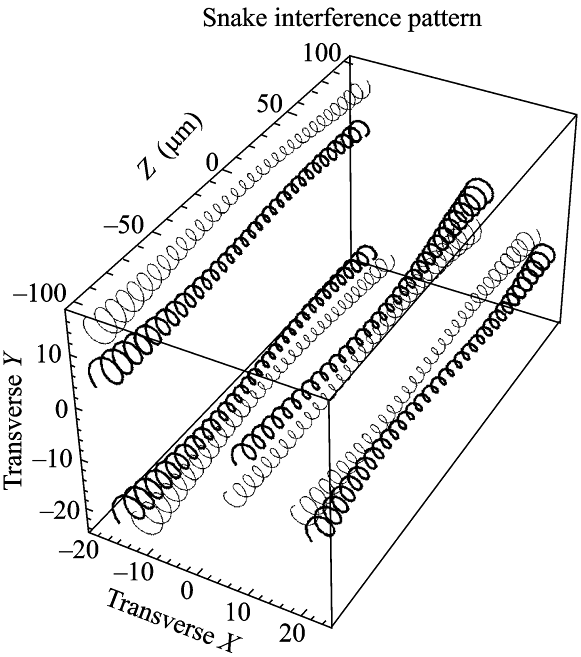

Fig. 3. Randomly spaced vortex–antivortex pair set in the volume of the stimulated Mandelstam–Brillouin scattering mirror. The interference of counterpropagating phase-conjugated waves with phase singularities forms a randomly spaced set of neighboring opposite-handed double helices. The right- and left-handed double helices are shown by the thick and thin curves, respectively. The typical scales are the same as in Fig. 2, but the transverse sizes and hands of the vortex lines vary smoothly along the z axis. Each element of the set (right- or left-handed double helix) rotates synchronously in one direction with the acoustic frequency. The optical-helix rotation is synchronized and has a high spatial correlation with the hypersonic helices.

charge) in Eq. (4) in comparison with the phase argument of the optical wavefunction.

The orbital angular momentum of an acoustic phonon admits the clear physical interpretation. The orbital angular momentum is due to the motion of the medium around a phase singularity. The existence of the acoustic angular momentum was recently proven experimentally for megahertz ultrasonic waves excited by a piezoelectric transducer array. Vortex structures in the acoustic-velocity field were directly detected with a hydrophone moved in the transverse plane [17]. In contrast to optical vortices [11], this measurement technique does not require the use of an additional reference wave. The generation of helical (singular) acoustic waves was also studied theoretically by the same authors using the Khokhlov–Zabolotskaya–Kuznetsov equation [18]

$$\frac{\partial^2 Q_{ac}(z, r, \phi, \tau)}{\partial z \partial \tau} = \Delta_\perp Q_{ac} + C_s(z, r, \phi)\frac{\partial^2 Q_{ac}}{\partial \tau^2} + \mu \frac{\partial^2 Q_{ac}^2}{\partial \tau^2}, \quad (7)$$

where $C_s(z, r, \phi)$ is the speed of sound dependent on the longitudinal coordinate and μ is the ratio of the Rayleigh (Fresnel) length $k_{p,s}D^2/2$ to the typical scale of the shock formation. It was shown that the results of numerical simulations are highly correlated with the experimentally measured acoustic fields [19]. Both the existence of single acoustic vortices and the vortex-pair interaction were proven. The repulsion of singularities with similar topological charges and the annihilation of acoustic singularities with opposite gyrations were observed in [20].

The presence of helical structures of radiation and hypersound (and, thus, refractive index) with a step equal to one-half of the optical wavelength $\lambda_{p,s}$ introduces significant corrections to the theory based on the approximation of slowly varying amplitudes [1]. Indeed, the wave equation in the initially homogeneous and isotropic dielectric allowing for electrostriction has the form [1]

$$\frac{\partial^2 \mathbf{E}(z, r, \phi, t)}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} + \Delta_\perp \mathbf{E} = \frac{1}{c^2 \epsilon_0}\frac{\partial^2 \mathbf{P}}{\partial t^2}, \quad (8)$$

where $\mathbf{P}$ is the electric-polarization vector (the dipole moment of unit volume), ϵ_0 is the permittivity of free space, and $\mathbf{E} = \mathbf{E}_p + \mathbf{E}_s$ is the total biharmonic field in the medium at the frequencies ω_p and ω_s. The equation for the pressure oscillations $p(z, r, \phi, t) \approx Q_{ac}$ in the acoustic wave excited by the striction force

$$f_{es} = \frac{\rho_0}{2\epsilon_0}\frac{\partial \epsilon}{\partial \rho}\nabla|\mathbf{E}|^2, \quad (9)$$

where ϵ is the relative permittivity of the medium and ρ_0 is the medium density in the absence of the field, is written as

$$\Delta p + \frac{2\Lambda}{v_{ac}^2}\Delta\frac{\partial p}{\partial t} - \frac{1}{v_{ac}^2}\frac{\partial^2 p}{\partial t^2} = \frac{\rho_0}{v_{ac}^2}\frac{\partial \epsilon}{\partial \rho}\Delta\frac{|\mathbf{E}|^2}{4\epsilon_0}, \quad (10)$$

where $v_{ac} = (\partial p/\partial \rho)_S^2$ is the speed of sound, $(\partial \rho/\partial p)_S$ is the adiabatic compressibility of the medium, and Λ is the sound damping. Even the initial nonparaxial equations of motion, Eqs. (8) and (10), show that the right-hand side comprising the interference force $|\mathbf{E}|^2$ describes a set of chaotically spaced "Archimedean-screw" pairs (Fig. 3) driving the rotation of the nonlinear medium around the phase singularities and ensuring the transfer of the orbital angular momentum from the radiation to the matter. Noteworthy, each vortex–antivortex pair also drives the matter in the opposite directions along the z axis (see Fig. 2).

In conclusion, the following fact should be noted. It is known that the Mandelstam–Brillouin nonlinearity is scalar since the acoustic pressure is modulated by the light intensity (see Eq. (10)) in the case of the electrostrictive mechanism of sound generation. Hence, the polarization state of the Stokes light $\mathbf{E}_s$ tends to copy the polarization state of the pump wave $\mathbf{E}_p$. As a result, the field configuration copying the polarization state [1] corresponds to the maximal growth rate [1] and the "polarization-state conjugation" of an optical field is achieved only using additional interferometric schemes [21]. However, according to the above analysis, the interference helical structures modulating the refractive index induce anisotropy in an initially isotropic dielectric [8]. In fact, the anisotropy of the medium and, hence, the absence of symmetry with respect to rotations, should result in the induced-birefringence effects, i.e., variation in the polarization state of the reflected wave $\mathbf{E}_s$ in comparison with the pump wave $\mathbf{E}_p$ even in the case of a perfect phase conjugation. The relation between the polarization of light and its spatial structure has been discussed for a long time in connection with the effects of photon spin–orbit interaction [22]. The induced-birefringence effect has recently also been studied for an ultracold-atom medium through which radiation with nonzero orbital angular momentum (e.g., a Laguerre–Gaussian laser beam) propagates [23].

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