Rotational Doppler shift of the phase-conjugated photon.

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The rotational Doppler shift of a photon with orbital angular momentum $\pm\ell\hbar$ is shown to be an even multiple of the angular frequency Ω of the reference frame rotation when photon is reflected from the phase-conjugating mirror. The one-arm phase-conjugating interferometer is considered. It contains N Dove prisms or other angular momentum altering elements rotating in opposite directions. When such interferometer is placed in the rotating vehicle the $\delta\omega = 4(N+1/2)\ell\cdot\Omega$ rotational Doppler shift appears. As a result the helical interference pattern will rotate with angular frequency $\delta\omega/2\ell$. The accumulation of angular Doppler shift via successive passages through the N image-inverting prisms is due to the phase conjugation, for conventional parabolic retroreflector the accumulation is absent. The features of such a vortex phase conjugating interferometry at the single photon level are discussed.

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1. Introduction.

Single photon interferometry utilizes the superposition of a mutually coherent quantum states Ψ_j [1] to whom photon belongs simultaneously. The interference pattern depends on a method of Ψ_j preparation. The double-slit Young interferometer creates two free-space wavefunctions Ψ_1,Ψ_2, whose interference pattern produced by detection of the individual photons is recorded by an array of detectors or a photographic plate located in the near or far field. In Mach-Zehnder configuration [2, 3] two wavefunctions separated by entrance beamsplitter recombine at the output beamsplitter. The Michelson interferometer recombines at the input beamsplitter two *retroflected* quantum states provided these states are phase-locked and their path difference δL is smaller than the coherence length L_c. Thus interference pattern is simply $\sim [1 + V(\delta L)\cdot\cos(k\cdot\delta L)]$, where $V(\delta L)$ is a visibility or second-order correlation function and $k = 2\pi/\lambda$. When retroflection is accompanied by wavefront reversal (PC) realized with phase-conjugating mirrors (PCM) [4] based upon Stimulated Brillouin scattering [5, 6], photorefractivity [7, 8] or holographic PCM's, the optical path δL difference is almost entirely compensated due to PC. Noteworthy the observable phase lag due to the relatively small frequency shift $\delta\omega = \omega_f - \omega_b$ arising due to the excitation of internal waves inside PCM volume [9], where ω_f and ω_b are the carrier frequencies of incident and PC-reflected photon respectively. This leads to the *frequency shift modulated* interference term $1 + V(\delta L)\cdot\cos(\delta k\cdot\delta L)$, where $\delta k = \delta\omega/c$ [6].

The aim of this article is to present the else nontrivial property of the phase conjugated optical vortex fields and their single photons. The predicted effect is in the accumulation of the small Doppler frequency shifts [3, 10, 11, 12, 13, 14] caused by a very slow rotations of the phase-conjugated interferometer as a whole and optical components therein. This effect takes place for the photon in the optical vortex quantum state [2,3] with topological charge ℓ, where the angular momentum $L_z = \pm\ell\cdot\hbar$ [15] is due to the phase singularity located at propagation axis z. Hereafter the spin component of angular momentum [16] is supposed to be zero due to the linear polarization. It is convinient to use the single-photon wavefunctions which coincide with the positive frequency component of the electric field envelope $|\mathbf{\Psi}> = \sqrt{2\epsilon_0}\cdot E(t,\vec{r})$ [17]. The square modulus of Ψ is proportional to the energy density of the *continuous wave* laser beams (CW) and to the photons count rate in a different fringes of the interference pattern for the single-photon experiments [18]. We will assume that Ψ is the Laguerre-Gaussian beam (LG) with $\ell\hbar$ orbital angular momentum (OAM) per photon [9] but any other isolated vortex solutions, e.g. Bessel vortices [20, 19] will led to the same results:

$$\mathbf{\Psi}_{(f,b)}(z,r,\theta,t) \sim \sqrt{2\epsilon_0}\cdot\frac{\exp[-i\omega_{(f,b)}t \pm ik_{(f,b)}z \pm i\ell\theta]}{(1+iz/z_R)}$$

$$E^0_{(f,b)}(r/D_0)^{|\ell|}\exp[-\frac{r^2}{{D_0}^2(1+iz/z_R)}], z_R = k_{(f,b)}{D_0}^2 \qquad (1)$$

where the cylindrical coordinates (z, r, θ) are used, D_0 is the vortex radius, z_R is Rayleigh range, Ψ_f, E_f stands for the forward wave, propagating in positive Z-direction, Ψ_b, E_b stands for the conjugated wave propagating in the opposite direction. Of special interest is the sub-Hz - order frequency splitting $\delta\omega/2\pi = c(k_f - k_b)/2\pi$ which appears due to the slow mechanical rotation of setup [3,21]. It was already shown that rotation of the $\lambda/2$ waveplate with angular frequency $\Omega \sim 2\pi(1-100)rad/s$ in a one arm of the Mach-Zehnder interferometer induces the rotational Doppler shift (RDS) $\delta\omega = 2\Omega\ell$ for circularily polarized broadband CW with linewidth $\Delta\omega/2\pi \simeq 10^{10}Hz$. In this configuration the broadband spectrum was shifted *as a whole* via mechanical rotation (by angular Doppler effect) at $\delta\omega/2\pi = \pm 2\cdot 7Hz$ and the beats at the output mirror induced an appropriate rotation of the interference pattern [22].